On the generalized motions/deformations of the floating bodies

Malenica S. & Bigot F.
Bureau Veritas, Paris, France

Introduction

For the sake of clarity, the compact matrix notations are introduced so that any vector quantity $\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$ is written as a column matrix $\{\boldsymbol{a}\}$ and to each vector quantity the skew symmetric matrix $[\boldsymbol{a}]$ is associated [5]. This allows writing the scalar product of two vectors $\{\boldsymbol{a}\}$ and $\{\boldsymbol{b}\}$ as $\boldsymbol{a}\cdot\boldsymbol{b} = \{\boldsymbol{a}\}^T\{\boldsymbol{b}\}$ and the vector product as $\boldsymbol{a}\wedge\boldsymbol{b} = [\boldsymbol{a}]\{\boldsymbol{b}\}$.
We refer to Figure1 and we define three coordinate systems. The coordinate systems (o,x,y,z) and (O,X,Y,Z) are both inertial coordinate systems fixed in space and are parallel to each other. The origin of the coordinate system (O,X,Y,Z) is located at the mean free surface with the Z axis being perpendicular to it. The coordinate system (G,x',y',z') is fixed to the body so that the two sets of coordinates (o,x,y,z) and (G,x',y',z') are related to each through the transformation matrix $[\boldsymbol{A}]$. With these notations any vector quantity $\{\boldsymbol{u}\}$ defined in (o,x,y,z) can be expressed as a function of its value in (G,x',y',z') by the following relation:

$$\{\boldsymbol{u}\} = [\boldsymbol{A}]\{\boldsymbol{u}'\} \tag{1}$$

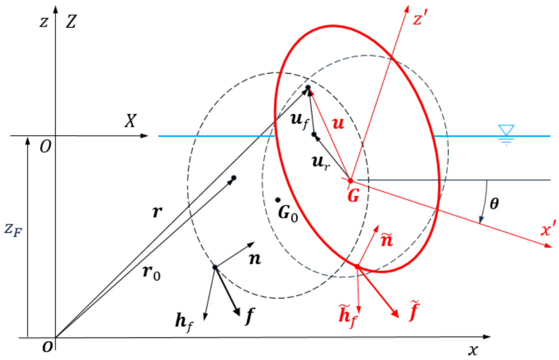

Figure 1: Generalized motion of the floating body.

Body dynamics

The total displacement $\{\boldsymbol{u}\}$ of the point attached to the body is decomposed into its global rigid body part $\{\boldsymbol{u}_r\}$ and its generalized deformation part $\{\boldsymbol{u}_f\}$ so that the instantaneous position in the earth fixed coordinate system becomes:

$$\{\boldsymbol{r}\} = \{\boldsymbol{r}_G\} + \{\boldsymbol{u}\} = \{\boldsymbol{r}_G\} + [\boldsymbol{A}]\{\boldsymbol{u}'\} = \{\boldsymbol{r}_G\} + [\boldsymbol{A}]\left(\{\boldsymbol{u}_r'\} + \{\boldsymbol{u}_f'\}\right) \tag{2}$$

In the general case the body deformation vector $\{\boldsymbol{u}_f'\}$ can be arbitrarily large but here the linear motion is assumed and the deformation vector is represented as a sum of the N_f modal contributions described by their space dependent mode shapes $\boldsymbol{h}_{fi}'(\boldsymbol{u}_r') = h_{fix'}'\boldsymbol{i}' + h_{fiy'}'\boldsymbol{j}' + h_{fiz'}'\boldsymbol{k}'$ and their time dependent modal amplitudes $\chi_{fi}(t)$. We write:

$$\{\boldsymbol{u}_f'(\boldsymbol{u}_r',t)\} = \sum_{i=1}^{N_f}\chi_{fi}(t)\{\boldsymbol{h}_{fi}'(\boldsymbol{u}_r')\} = [\boldsymbol{\hbar}_f']\{\chi_f\} \tag{3}$$

where $[\boldsymbol{\hbar}_f']$ is the $3\times N_f$ matrix which columns contain the 3 components of the mode shape vectors and $\{\chi_f\}$ is the vector of the corresponding modal amplitudes. With these notations the instantaneous position vector of the point attached to the body, becomes:

$$\{\boldsymbol{r}\} = \{\boldsymbol{r}_G\} + [\boldsymbol{A}]\left(\{\boldsymbol{u}_r'\} + [\boldsymbol{\hbar}_f']\{\chi_f\}\right) \tag{4}$$

It is important to note that the instantaneous position of the center of gravity is obtained by considering the rigid body motions only and that the elastic modes are defined in the local coordinate system (G,x',y',z').
The application of the D'Alambert principle of the virtual work within the Lagrangian formalism leads to the following linear dynamic motion equation of the flexible floating body [5]:

$$\begin{bmatrix} [\boldsymbol{m}] & [0] & [0] \\ [0] & [\boldsymbol{I}_{\theta\theta}'] & [\boldsymbol{I}_{\theta f}'] \\ [0] & [\boldsymbol{I}_{\theta f}']^T & [\boldsymbol{I}_{ff}'] \end{bmatrix} \begin{Bmatrix} \{\boldsymbol{a}_G\} \\ \{\dot{\boldsymbol{\Omega}}'\} \\ \{\ddot{\chi}_f\} \end{Bmatrix} = \begin{Bmatrix} \{\boldsymbol{F}\} \\ \{\boldsymbol{M}'\} \\ \{\boldsymbol{F}_f\} - [\boldsymbol{K}]\{\chi_f\} \end{Bmatrix} \tag{5}$$

where $[\boldsymbol{K}]$ is the structural stiffness matrix for the flexible modes, if existing.
We note that the motion equation is expressed in the body fixed coordinate system, the mass matrix $[\boldsymbol{m}]$ is the diagonal matrix with the elements equal to body mass and other inertia terms are given by the following expressions:

$$[\boldsymbol{I}'_{\theta\theta}] = \iiint_{\forall_B} [\boldsymbol{u}']^T[\boldsymbol{u}']dm \quad , \quad [\boldsymbol{I}'_{\theta f}] = \iiint_{\forall_B} [\boldsymbol{u}'_r]^T[\boldsymbol{\hbar}'_f]dm \quad , \quad [\boldsymbol{I}'_{ff}] = \iiint_{\forall_B} [\boldsymbol{\hbar}'_f]^T[\boldsymbol{\hbar}'_f]dm \tag{6}$$

We note that the above equation of motion is valid for arbitrarily large rigid body motions and small flexible motions. The external forces are assumed to act at the discrete points on the body and we denote each of them by $\{\tilde{\boldsymbol{f}}'_j\}$. In the present context of the freely floating flexible body, it exist two types of the external forces: the gravity and the pressure forces:

$$\{\tilde{\boldsymbol{f}}'_j\} = \{\tilde{\boldsymbol{f}}^{g\prime}_j\} + \{\tilde{\boldsymbol{f}}^{p\prime}_j\} \tag{7}$$

We note that the gravity force remains constant in the earth fixed coordinate system and the discrete pressure forces are obtained by integrating the pressure over the element of the wetted body surface, and we can write:

$$\{\tilde{\boldsymbol{f}}^{g\prime}_j\} = -m_j g[\boldsymbol{A}]^T\{\boldsymbol{k}\} \quad , \quad \{\tilde{\boldsymbol{f}}^{p\prime}_j\} = -\varrho g P\{\tilde{\boldsymbol{n}}'\}dS_j \tag{8}$$

where P is the external pressure and $\{\tilde{\boldsymbol{n}}'\}dS_j$ is the oriented element of the wetted surface.
With these notations, the instantaneous rigid body forces, rigid body moments and the generalized modal forces become:

$$\{\boldsymbol{F}'\} = \sum_{j=1}^{N}\{\tilde{\boldsymbol{f}}'_j\} \quad , \quad \{\boldsymbol{M}'\} = \sum_{j=1}^{N}[\boldsymbol{u}']\{\tilde{\boldsymbol{f}}'_j\} \quad , \quad F_{fi} = \sum_{j=1}^{N}\{\tilde{\boldsymbol{h}}'_{fi}\}^T\{\tilde{\boldsymbol{f}}'_j\} \tag{9}$$

Linearization

The first step in the linearization procedure is to assume the rigid body motions to be small i.e. of the same order as the flexible ones which basically means that the transformation matrix $[\boldsymbol{A}]$ is developed in the following form:

$$[\boldsymbol{A}] = 1 + [\boldsymbol{\theta}] \tag{10}$$

In parallel the Taylor series expansion is used to express the instantaneous value of any physical quantity, scalar $\tilde{q}'$ or vector $\{\tilde{\boldsymbol{q}}'\}$, as a function of its value at rest. We write:

$$\tilde{q}' = \left(1 + \boldsymbol{u}'_f\nabla\right)q' \quad , \quad \{\tilde{\boldsymbol{q}}'\} = \left(1 + \boldsymbol{u}'_f\nabla\right)\{\boldsymbol{q}'\} \quad , \quad \boldsymbol{u}'_f\nabla = u'_{fx'}\frac{\partial}{\partial x'} + u'_{fy'}\frac{\partial}{\partial y'} + u'_{fz'}\frac{\partial}{\partial z'} \tag{11}$$

In particular this means that we can write for the mode shape vector $\{\tilde{\boldsymbol{h}}'_{fi}\}$ the following development:

$$\{\tilde{\boldsymbol{h}}'_{fi}\} = \left(1 + \boldsymbol{u}'_f\nabla\right)\{\boldsymbol{h}'_{fi}\} = \{\boldsymbol{h}'_{fi}\} + [\nabla\boldsymbol{h}'_{fi}]\{\boldsymbol{u}'_f\} \tag{12}$$

Furthermore, in order to describe the instantaneous body deformations in a convenient form it is convenient to introduce the notion of the deformation gradient $[\boldsymbol{F}]$ as follows:

$$[\boldsymbol{F}] = [\boldsymbol{I}] + [\nabla\boldsymbol{u}'_f] \tag{13}$$

where $[\nabla\boldsymbol{u}'_f]$ denotes the displacement gradient tensor.
This allows writing the instantaneous normal vector in the following form:

$$\{\tilde{\boldsymbol{n}}'\}dS = \|\boldsymbol{F}\|([\boldsymbol{F}]^{-1})^T\{\boldsymbol{n}'\}dS = \left(1 + \nabla\boldsymbol{u}'_f - [\nabla\boldsymbol{u}'_f]^T\right)\{\boldsymbol{n}'\}dS + O\left(\boldsymbol{u}'^2_f\right) \tag{14}$$

Generalized description of the body motions/deformations

In the linear case, it is convenient to rewrite the rigid body motions in the generalized modal form. We write:

$$\{\boldsymbol{R}_G\} + [\boldsymbol{\theta}]\{\boldsymbol{u}'_r\} = \sum_{i=1}^{6}\chi^r_i(t)\{\boldsymbol{h}'_{ri}(\boldsymbol{u}'_r)\} \quad , \quad \{\boldsymbol{r}\} = \{\boldsymbol{r}_0\} + [\boldsymbol{\hbar}']\{\boldsymbol{\xi}\} \tag{15}$$

where $\{\boldsymbol{R}_G\}$ denotes the translation of the center of gravity and the matrix $[\boldsymbol{\hbar}']$ is the global mode shape matrix with the dimensions $3 \times \left(6 + N_f\right)$ where the first six columns contain the rigid body modes and the remaining columns contain the deformable modes. Similarly, the vector $\{\boldsymbol{\xi}\}$ is the vector of the modal amplitude where the first 6 elements represents the rigid body translations and the rotations, and the remaining elements are the amplitudes of the deformation modes.

Hydrostatic restoring

Once the velocity potential calculated, the evaluation of the linear dynamic pressure forces is rather straightforward, and here below we concentrate on the hydrostatic restoring part which is induced by the hydrostatic pressure and the gravity. After some manipulations the following expressions are obtained:

$$\mathcal{F}^{hs}_i = -\varrho g\iint_{S_B}\left[\zeta_v\{\boldsymbol{h}'_i\}^T + Z\left(\{\boldsymbol{h}'_i\}^T\left(\nabla\boldsymbol{u}'_f - [\nabla\boldsymbol{u}'_f]^T\right) + \{\boldsymbol{u}'_f\}^T[\nabla\boldsymbol{h}'_i]^T\right)\right]\{\boldsymbol{n}'\}dS$$

$$\mathcal{F}^{g}_i = \sum_{j=1}^{N_m} -m_j g\left(\{\boldsymbol{h}'_i\}^T[\boldsymbol{\theta}] + \{\boldsymbol{u}'_f\}^T[\nabla\boldsymbol{h}'_i]^T\right)\{\boldsymbol{k}\}$$

where N_m is the total number of the mass points, and ζ_v denotes the vertical displacement of the point attached to the body:

$$\zeta_v = \sum_{i=1}^{6+N_f} \xi_i \{\boldsymbol{h}'_i\}^T \{\boldsymbol{k}\} = \sum_{i=1}^{6+N_f} \xi_i h'_{iz'} \tag{16}$$

The elements of the restoring matrix can be deduced in the form:

$$C_{ij}^{hs} = -\varrho g \begin{cases} \iint_{S_B} h'_{jz'} \{\boldsymbol{h}'_i\}^T \{\boldsymbol{n}'\} dS & , \quad j = 1,6 \\ \iint_{S_B} \left[h'_{jz'} \{\boldsymbol{h}'_i\}^T + Z \left(\{\boldsymbol{h}'_i\}^T \left(\nabla \boldsymbol{h}'_j - [\nabla \boldsymbol{h}'_j]^T \right) + \{\boldsymbol{h}'_j\}^T [\nabla \boldsymbol{h}'_i]^T \right) \right] \{\boldsymbol{n}'\} dS & , \quad j > 6 \end{cases} \tag{17}$$

$$C_{ij}^{g} = \sum_{n=1}^{N} m_n g \{\boldsymbol{k}\}^T \begin{cases} [\nabla \boldsymbol{h}'_j] \{\boldsymbol{h}'_i\} & , \quad j = 1,6 \\ [\nabla \boldsymbol{h}'_i] \{\boldsymbol{h}'_j\} & , \quad j > 6 \end{cases} \tag{18}$$

There has been lot of discussions in the past concerning the correct expression for the hydrostatic restoring of the deformable bodies. Some discussions related to this work were presented in [2]. In that paper exactly the same formulation was obtained by three very different methods, but some problems were reported regarding its application to the evaluation of the internal loads. The fundamental difference in between the present formulation and the formulations proposed in [2] is related to the proper accounting for the coordinate systems in which the body deformations are defined and in which the forces are expressed. The main point is that both the normal vector and the mode shape vector do not change in the body fixed coordinate system while the gravity force vector does, and this fact seems to not be properly accounted for in [2]. We also note that some other formulations for hydrostatic restoring were proposed in the literature [3,4,6] but we do not discuss theme here even if some of them give the results for the internal loads which are similar to the present formulation.

Numerical results

The case of the hyper elastic barge is chosen. The detailed description of the model is presented in [3] and here we just briefly present the results for the RAO of the horizontal bending moment in oblique waves. The results clearly shows the difference in between the two approaches and demonstrates the validity of the new approach.

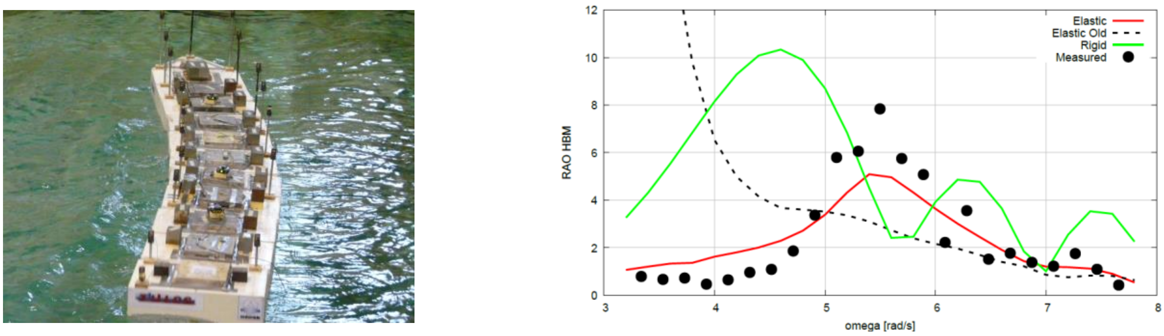

Figure 2: Hyper elastic barge in oblique waves and the RAO's of the horizontal bending moment at midship section. (Elastic Old denotes the results obtained using the old formulation for hydrostatic restoring)

In order to demonstrate the elegance of the generalized modal approach the case of the crane operations is considered. We refer to Figure 3 where the basic configuration is presented.

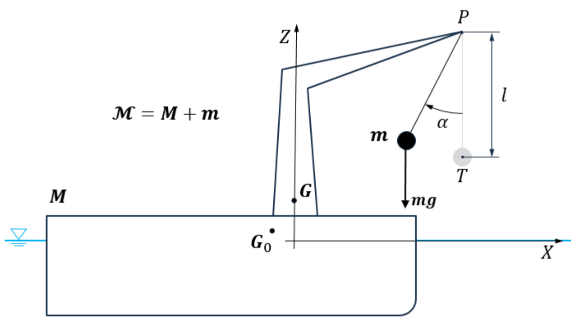

Figure 3: Floating body with attached pendulum.

A more classical way of solving the problem was described in [1] where the multibody interaction combined with constraint equations was used. This leads to rather complex final expressions for the dynamic motion equation. Within the generalized modal approach the description of the problem becomes extremely simple. Indeed, the overall problem is formulated as a single body problem with 8 degrees of freedom: 6 overall rigid body modes of motion (with pendulum mass included and treated as fixed) and 2 rotational modes describing the motion of the pendulum mass (with amplitudes α and γ). The definition of the rigid body mode shapes is the classical one and the pendulum mode shapes are defined by:

$$\{\boldsymbol{h}_7'\} = [\boldsymbol{j}'](\{\boldsymbol{u}_r'\} - \{\boldsymbol{u}_P'\}) = -l\{\boldsymbol{i}'\} \quad , \quad [\nabla \boldsymbol{h}_7'] = [\boldsymbol{j}']$$
$$\{\boldsymbol{h}_8'\} = [\boldsymbol{i}'](\{\boldsymbol{u}_r'\} - \{\boldsymbol{u}_P'\}) = l\{\boldsymbol{j}'\} \quad , \quad [\nabla \boldsymbol{h}_8'] = [\boldsymbol{i}']$$

The direct application of the above described generalized modal method leads to very few modifications of the classical rigid body mechanics. The only changes concern the inertia and the hydrostatic restoring matrix which changes as follows:

$$[\boldsymbol{\mathcal{M}}] = \begin{bmatrix} & & & & & & -lm & 0 \\ & & & & & & 0 & lm \\ & & & & & & 0 & 0 \\ & & & \mathcal{M}_{ij} & & & 0 & -lmz_{TG} \\ & & & & & & -lmz_{TG} & 0 \\ & & & & & & lmy_{TG} & lmx_{TG} \\ -lm & 0 & 0 & 0 & -lmz_{TG} & lmy_{TG} & l^2m & 0 \\ 0 & lm & 0 & -lmz_{TG} & 0 & lmx_{TG} & 0 & l^2m \end{bmatrix}, \quad [\boldsymbol{\mathcal{C}}] = g\begin{bmatrix} & & & & & & 0 & 0 \\ & & & & & & 0 & 0 \\ & & & & & & 0 & 0 \\ & & & \mathcal{C}_{ij} & & & 0 & lm \\ & & & & & & lm & 0 \\ & & & & & & 0 & 0 \\ 0 & 0 & 0 & 0 & lm & 0 & lm & 0 \\ 0 & 0 & 0 & lm & 0 & 0 & 0 & lm \end{bmatrix}$$

where $\mathcal{M}_{ij}$ and $\mathcal{C}_{ij}$ are the classical inertia and hydrostatic restoring matrices with the pendulum mass considered as fixed. The remaining hydrodynamic coefficients (added mass, damping and excitation) are non-zero for the first 6 modes only and they are evaluated exactly in the same way as for the single rigid body case. Two type of methods i.e. the classical one (denoted Old) and the generalized modal one (denoted New) are compared in Figure 4 for a typical case. We can observe a typical effect of the pendulum and we can also see that the two classes of results are identical.

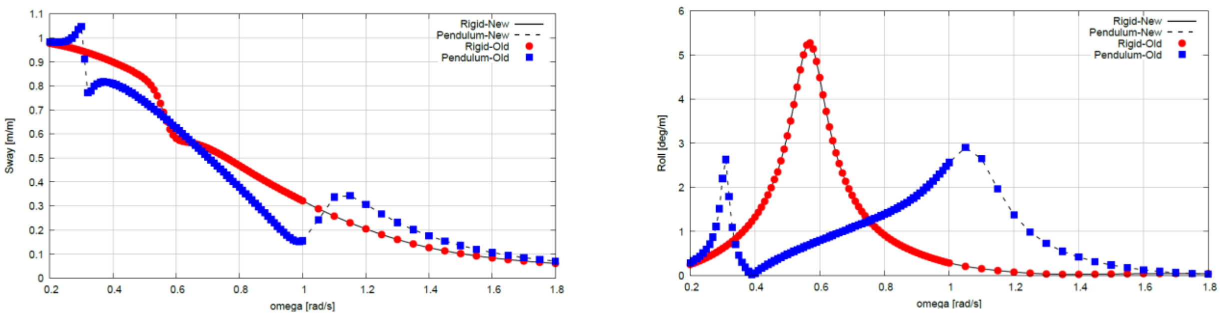

Figure 4: Sway (left) and roll (right) motion of the floating body with attached pendulum.

Discussions & conclusions

We have discussed here a so called generalized modal approach for solving the arbitrarily linear dynamics of the deformable floating body. New formulation for the hydrostatic restoring was proposed and validated so that some inconsistencies from the past are corrected. Furthermore, it was shown that the generalized modal approach is not limited to flexible bodies but represents a very elegant and general method for solving the different mechanical problems in seakeeping. This was demonstrated on the case of lifting operations and it was shown that very simple modifications of the existing seakeeping code are necessary to properly solve the problem. The critical point in the analysis appears to be the correct definition of the different mode shapes and the evaluation of the corresponding generalized inertia and hydrostatic restoring matrices.

Acknowledgements

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References

[1] Malenica S., Orozco J.M. & Chen X.B., 2005. : “Some aspects of seakeeping of the floating body with attached pendulum”, 11th IMAM Conf., Lisbon, Portugal.
[2] Malenica S., Molin B., Tuitman J.T., Bigot F. & Senjanovic I., 2009.: “Some aspects of hydrostatic restoring for elastic bodies”, 24th IWWWFB, Saint Petersburg, Russia
[3] Maherault S, Derbanne Q. & Bigot F., 2013.: “Fatigue damage calculation of ULCS based on hydroelastic model for springing”, Ships and Offshore Structures, DOI:10.1080/17445302.2012.750087
[4] Riggs H.R., 2009. : "The hydrostatic stiffness of flexible floating structure for linear hydroelasticity", Journal of Offshore Mechanics and Arctic Engineering, Vol 131.
[5] Shabana A.A.1989.: "Dynamics of multibody systems", John Wiley & Sons
[6] Senjanovic I., Vladimir N. & Tomic M., 2012. : "Formulation of consistent restoring stiffness in ship hydroelastic analysis", J. Eng. math, Vol.72, pp. 141-157.