Oblique impact of elastic plate on thin liquid layer

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The unsteady, two-dimensional and nonlinear problem of oblique impact by an elastic plate onto a thin liquid layer is considered. The edges of the plate are free of bending stresses and shear forces. The present study was motivated by the experiments on droplets deposition in annular gas-liquid flow and mass exchange between the gas core and the liquid film [1]. Bubbles entrapped in the liquid film were observed for some conditions of oblique droplet impact onto the film. The rate of bubble entrainment is high and cannot be explained by the air-cushion effect which is less significant for oblique liquid impacts, see [2, 3]. The present study aims at identifying some possible mechanisms of bubble entrainment in a thin liquid layer as a result of oblique impact onto the layer by a deformable body, which mimics a liquid droplet. The simplest configuration with free-free elastic thin plate is considered. The air can be trapped (a) in front of the impacting body due to the body deformation towards the liquid and jetting, (b) under the body because of the body vibration and cavitation caused by the impact, and (c) behind the body because of complex and oscillatory behaviour of the wake. These potential mechanisms of air entrainment are important also at larger scale of high-speed boats and ditching aircraft.

We assume that the penetration of the body into the liquid layer is comparable with the thickness of the layer and the horizontal dimension of the problem, which is the projection of the plate onto the layer, is much greater than the layer thickness. These assumptions make it possible to describe the flow between the impacting body and the bottom of the liquid layer within the thin-layer approximation, see [4, 5], using certain matching conditions at the edges of the wetted part of the body surface. In the two-dimensional problem of oblique plate impact, the matching condition at the leading edge of the wetted part of the plate surface describes the jetting there, see Figure 1, and the matching condition at the trailing edge describes the liquid separation from the plate surface. The liquid separation may occurs either from the left edge of the plate, see Figure 1, or from the inner part of the smooth plate surface. In the later case, the position of the separation point is determined using the Brillouin-Villat condition [6]. The positions of both leading and trailing edges of the wetted part of the plate depends on the plate motions and elastic deflection. This problem was studied in [7] for deep water within linear hydrodynamic model and in [8] for shallow water and a rigid plate.

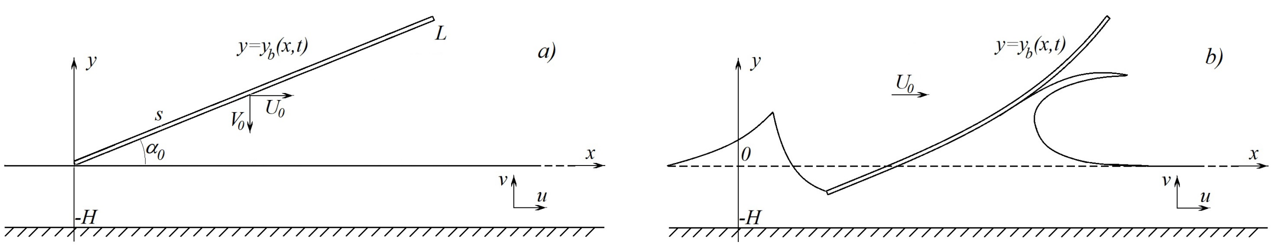

Fig. 1 Oblique impact by elastic plate: (a) Initial position of the plate and notation, (b) Scheme of the flow and plate deflection during impact. Here $y = y_b(x,t)$ is the position of the plate in the global coordinate system, $y_b(x_p(s,t),t) = y_p(s,t)$.

The motion of the plate is described in a parametric form $x = x_p(s,t)$ and $y = y_p(s,t)$, where s is the coordinate along the plate, $0 < s < L$, and t is the time, $t > 0$. We are concerned with such conditions of oblique impact that both the vertical rigid displacement of the plate and its elastic deflection are of the order of the fluid depth H,

$$y_p = H\tilde{y}_p(\tilde{s},\tilde{t}),\ x_p = L(\tilde{s}+\lambda\tilde{t}+O(\varepsilon^2)),\ s = L\tilde{s},\ t = H\tilde{t}/V_0,\ \varepsilon = H/L,\ \alpha_0 = \varepsilon\tilde{\alpha}_0,\ \alpha = \varepsilon\tilde{\alpha}(\tilde{t}), \quad (1)$$

where the dimensionless variables are denoted by tilde, and ε is a small parameter of the problem. The parameters $\lambda = U_0\varepsilon/V_0$ and $\tilde{\alpha}_0$ are assumed of order $O(1)$. Note that we do not separate rigid and elastic motions of the plate and can neglect the variation of the horizontal speed of the plate in the leading order for small ε.

The vertical displacement of the plate, $\tilde{y}_p(\tilde{s},\tilde{t})$, in the dimensionless variables and in the leading order as $\varepsilon \to 0$ is described by the equations,

$$M\frac{\partial^2\tilde{y}_p}{\partial\tilde{t}^2} + \frac{V_*^2}{V_0^2}\frac{\partial^4\tilde{y}_p}{\partial\tilde{s}^4} = \tilde{p}(\tilde{s},\tilde{t}) - \mathrm{Fr}^{-2}(\varepsilon^2\tilde{y}_p - M) \qquad (0<\tilde{s}<1,\ \tilde{t}>0), \quad (2)$$

$$\frac{\partial^2\tilde{y}_p}{\partial\tilde{s}^2} = 0, \quad \frac{\partial^3\tilde{y}_p}{\partial\tilde{s}^3} = 0 \quad (\tilde{s}=0,1), \qquad \tilde{y}_p = \tilde{s}\tilde{\alpha}(0), \quad \frac{\partial\tilde{y}_p}{\partial\tilde{t}} = -1\ (\tilde{t}=0), \quad (3)$$

where $M = m\varepsilon^2/(\rho H)$, Fr $=V_0/\sqrt{gH}$ and $V_*^2 = EJH\varepsilon^2/(\rho L^4)$. Here V_* is a characteristic speed, which depends on elastic properties of the plate. For an aluminium plate with density of aluminium $\rho_p = 2700$ kg/m^3 and the Young modulus $E = 68.3\cdot 10^9$ N/m^2, of length $L = 10$ cm and thickness 2 mm, and water layer of thickness 1 cm, we find $V_* = 0.2134$ m/s. The interaction of these elastic plate with the thin layer of water is strong for the vertical impact speed V_0 being of order V_*. For the same values of the impact parameters, we find $M = 0.0054$ and $\mathrm{Fr} = V_*/\sqrt{gH} \approx 0.68$. The dimensional hydrodynamic pressure, $p(s, y_p(s,t), t)$, along the wetted plate surface is scaled as $\rho V_0^2\varepsilon^{-2}\tilde{p}(\tilde{s},\tilde{t}) - \rho g H\tilde{y}_p(\tilde{s},\tilde{t})$. The horizontal velocity of the flow under the plate is scaled as $u(s,t) = U_0 + V_0\varepsilon^{-1}\tilde{U}(\tilde{s},\tilde{t})$. The tilde is dropped below.

The liquid flow under the wetted part of the plate is approximately described by the one-dimensional equations of thin layer flow, see [6],

$$\frac{\partial Y}{\partial t} + \frac{\partial}{\partial s}\Big(U(s,t)Y(s,t)\Big) = 0, \qquad \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial s} = -\frac{\partial p}{\partial s}, \quad (4)$$

where $Y(s,t) = 1 + y_p(s,t)$ is the dimensionless thickness of the liquid layer. Equations (4) should be solved in the region $c_L(t) < s < c_R(t)$, where $s = c_L(t)$ corresponds to the left edge of the wetted part of the plate, and $s = c_R(t)$ is the coordinate of the leading edge of the wetted surface. We assume that the leading edge propagates in the global coordinate system at a speed much greater than the critical speed of the thin liquid layer, $\sqrt{gH}$. In this case, the liquid in front of the moving plate is not disturbed. The rest state, $p = 0$, $U = -\lambda$, in $s > c_R(t)$, is matched to the unsteady one-dimensional flow under the plate, which is described by equations (4), by a two-dimensional quasi-steady jet solution [6]. The matching leads to the following conditions for the speed of the leading edge in the moving coordinate system and the hydrodynamic pressure at this edge,

$$\frac{dc_R}{dt} = \frac{U_R(t)\sqrt{Y_R(t)}}{2(\sqrt{Y_R(t)}-1)} + \frac{\lambda}{2}\,\frac{2-\sqrt{Y_R(t)}}{\sqrt{Y_R(t)}-1}, \qquad p_R(t) = \frac{(\lambda+U_R(t))^2}{2(\sqrt{Y_R(t)}-1)}, \quad (5)$$

where $p_R(t) = p(c_R(t),t)$, $U_R(t) = U(c_R(t),t)$ and $Y_R(t) = 1 + y_p(c_R(t),t)$. Initially $Y_R(0) = 1$ and the speed of the leading contact point, dc_R/dt, is finite only if $U_L(0) = -\lambda$, which corresponds to the uniform flow in front of the plate in the moving system. The trailing edge, $s = c_L(t)$, where the liquid separates from the plate, is initially at the left edge of the plate, $c_L(t) = 0$, where $p(0,t) = 0$ and the pressure is positive (above the ambient pressure) in front of this edge, $p_s(0,t) > 0$. The trailing edge starts moving from the left edge of the plate when $p_s(0,t)$ drops to zero. Then the position of the separation point is determined using the Brillouin-Villat condition, $p(c_L(t),t) = 0$ and $p_s(c_L(t),t) = 0$.

The plate equation (2) is integrated by the normal mode method [7], where the shape function $y_p(s,t)$ is sought in the form

$$y_p(s,t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(s), \quad (6)$$

with $\psi_n(s)$ being the orthonormal normal modes of the dry elastic plate with free-free edges, and $a_n(t)$

are the coefficients to be determined as the solutions of the following system of ordinary differential equations,

$$M\ddot{a}_k + \mu\lambda_k^4 a_k = \int\limits_{c_L(t)}^{c_R(t)} p(s,t)\psi_k(s)\,ds - \varepsilon^2\,\mathrm{Fr}^{-2}\int\limits_{c_L(t)}^{c_R(t)} y_p(s,t)\psi_k(s)\,ds + M\,\mathrm{Fr}^{-2}\delta_{k1} \qquad (7)$$

where $\delta_{11} = 1$ and $\delta_{k1} = 0$ for $k > 1$, λ_k is the spectral parameter corresponding to the k-th mode. Initial conditions for the system (7) are

$$a_1(0) = \frac{1}{2}\alpha_0, \quad \dot{a}_1(0) = -1, \quad a_2(0) = \frac{\alpha_0}{2\sqrt{3}}, \quad \dot{a}_2(0) = 0, \quad a_k(0) = \dot{a}_k(0) = 0 \quad (k \geq 3). \qquad (8)$$

Equations (4) are integrated in s,

$$U(s,t) = \frac{Y_R(t)}{Y(s,t)}U_R(t) + \frac{1}{Y(s,t)}\int\limits_{s}^{c_R(t)} y_{p,t}(s_0,t)\,ds_0, \qquad (9)$$

$$p(s,t) = p_R(t) + \frac{1}{2}\Big(U_R^2(t) - U^2(s,t)\Big) + \int\limits_{s}^{c_R(t)} U_t(s_0,t)\,ds_0. \qquad (10)$$

Using equations (6), (9), (10) and the edge conditions at $s = c_L(t)$ and $s = c_R(t)$, the integrals in (7) are evaluated as

$$\int\limits_{c_L(t)}^{c_R(t)} y_p(s,t)\psi_k(s)\,ds = \sum_{n=1}^{\infty} a_n(t)R_{nk}(t),\ R_{nk}(t) = \int\limits_{c_L(t)}^{c_R(t)} \psi_n(s)\psi_k(s)\,ds,\ \bar{\psi}_k(c_L,s) = \int\limits_{c_L}^{s}\psi_k(s_0)\,ds_0, \quad (11)$$

$$\int\limits_{c_L(t)}^{c_R(t)} p(s,t)\psi_k(s)\,ds = \sum_{m=1}^{\infty}\ddot{a}_m(t)B_{mk}(t) + \int\limits_{c_L}^{c_R} U^2(s,t)\left\{\frac{Y_s(s,t)}{Y(s,t)}\Big[\bar{\psi}_k(s,c_L) - \frac{\nu_k}{\nu_0}\Big] - \psi_k(s)\right\}ds$$

$$+U_L^2\frac{\nu_k}{\nu_0} + \Big(p_R + U_R^2\Big)\Big(R_{1k} - \frac{\nu_k}{\nu_0}\Big),\ B_{mk} = \frac{\nu_m\nu_k}{\nu_0} - \int\limits_{c_L}^{c_R}\frac{\bar{\psi}_m(c_L,s)\bar{\psi}_k(c_L,s)}{Y(s,t)}\,ds,\ \nu_k(t) = \int\limits_{c_L}^{c_R}\frac{\bar{\psi}_k(c_L,s)\,ds}{Y(s,t)}.$$

The results shown in Figures 2-5 are for the oblique impact of an elastic plate of length $L = 10$ cm and thickness 2 mm made of a material with density $\rho = 2670$ kg/m^3. Initial horizontal and vertical speeds of the plate are $U_0 = 5$ m/s and $V_0 = 5$ m/s. The water depth is $H = 2$ cm. Calculations were performed with 2 rigid and 7 elastic modes of the plate. The results with 7 and 5 elastic modes are almost identical. The truncated system of nonlinear equations (5)-(11) was integrated in time numerically by the Euler method with the dimensionless time step 2×10^{-4}. The position of the trailing edge $c_L(t)$ was determined by the secant method with accuracy 10^{-10}. Case *a)* corresponds to $E = 68 \times 10^6$ Pa and the initial inclination angle $\alpha_0 = 3^o$. In this case, the plate is totally wetted in 1 ms. The liquid separates from the left edge of the plate, $c_L(t) = 0$. Case *b)* corresponds to $E = 68 \times 10^7$ Pa and the initial inclination angle $\alpha_0 = 10^o$. In this case, the calculations stop when the right edge of the plate touches the liquid surface with trapping an air cavity. The separation point is under the plate, $c_L(t) > 0$. The impact stage is three times longer than in case a) with nonuniform pressure in the contact region.

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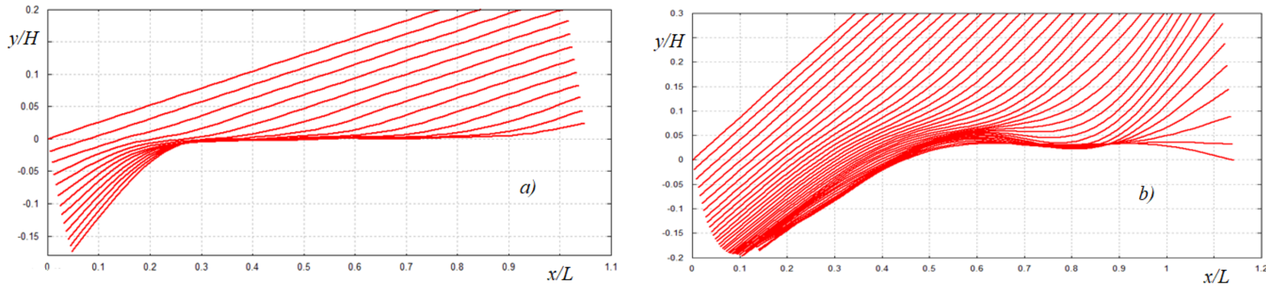

Fig. 2 Positions of the plate at different time instants for a soft plate (a) and more rigid plate (b).

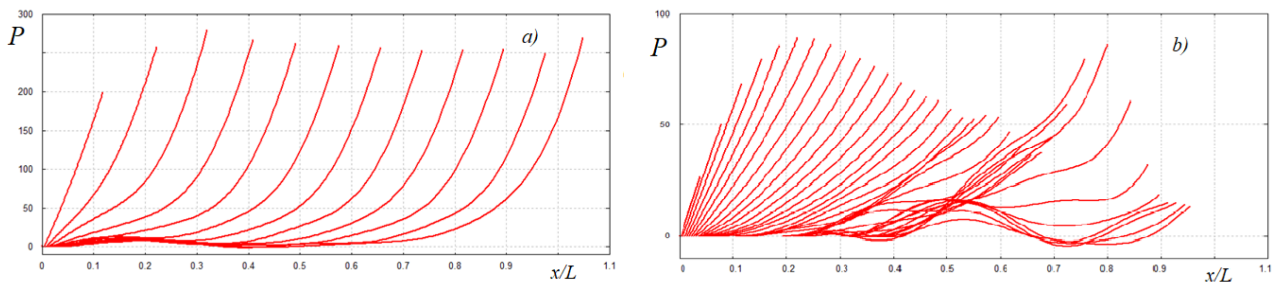

Fig. 3 Pressure distributions (in kPa) along the wetted part of the plate at different time instants.

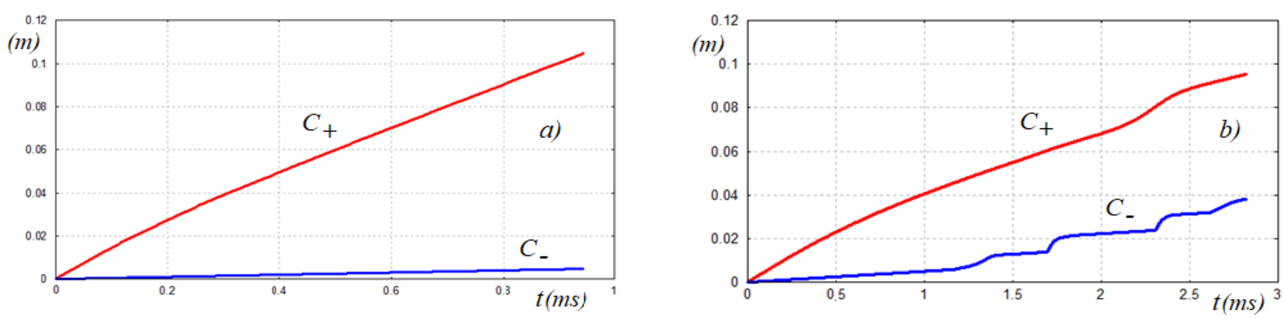

Fig. 4 The x-coordinates of the right, $C_+ = Lc_R(t) + U_0t$, and left, $C_- = Lc_L(t) + U_0t$, edges of the contact region.

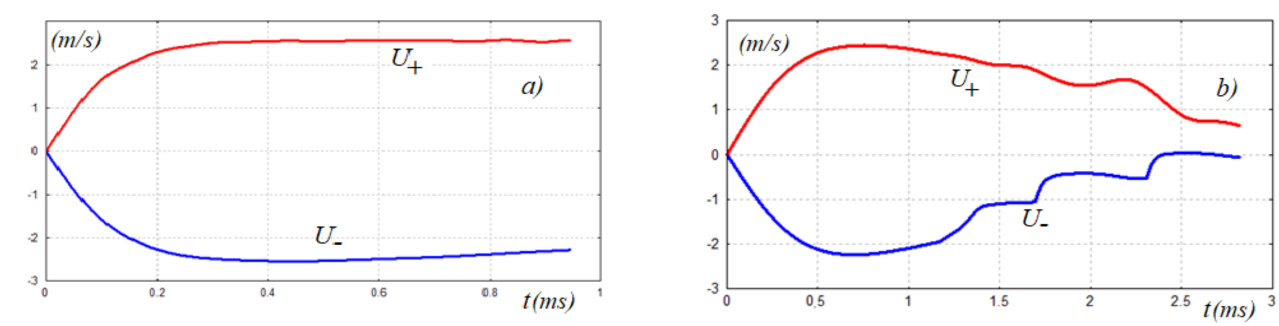

Fig. 5 The horizontal velocities of the global flow under the plate at the right, $U_+(t)$, and left, $U_-(t)$, edges of the contact region.